1. WAVE EQUATION & PARAMETERS

General form (travelling wave):

$$y(x,t) = A\sin(\omega t - kx + \phi_0)$$

- A = amplitude [m]
- $\omega = 2\pi f = 2\pi/T$ = angular frequency [rad/s]
- $k = 2\pi/\lambda$ = wave number [rad/m]
- ϕ_0 = initial phase
- $v = f\lambda = \omega/k$ [m/s]

$-x$ direction: $y = A\sin(\omega t + kx)$

Particle velocity: $v_p = \partial y/\partial t = A\omega\cos(\omega t - kx)$
Max particle speed: $v_{p,\max} = A\omega$
Particle acceleration: $a_p = -\omega^2 y$
Wave speed shortcut:

$$v_{\text{wave}} = -\frac{\partial y/\partial t}{\partial y/\partial x}$$

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- $v_{\text{wave}} \neq v_{\text{particle}}$; wave speed is a medium property, particle speed depends on A and ω
- Phase of wave at x: $(\omega t - kx)$ — larger x means *lagging* phase for $+x$ wave
- $a_p = -\omega^2 y$ is **SHM**: each particle oscillates as SHM

2. SPEED OF WAVES

(A) String (Transverse):

$$v = \sqrt{T/\mu}$$

T=tension [N], $\mu = m/L$=linear mass density [kg/m]

(B) Sound in Gas (Newton–Laplace):

$$v = \sqrt{\gamma P/\rho} = \sqrt{\gamma RT/M}$$

$\gamma = C_p/C_v$, P=pressure, ρ=density, M=molar mass
Air at 0°C: $v_0 \approx 332$ m/s
Temp. correction: $v_T = v_0\sqrt{T/273} \approx 332 + 0.61\,t$ (t in °C)
Pressure effect: None at constant T (since P/ρ = const)
Humidity: $v_{\text{moist}} > v_{\text{dry}}$ ($\rho_{\text{moist}} < \rho_{\text{dry}}$)

(C) In liquid: $v = \sqrt{B/\rho}$ **(D) In rod:** $v = \sqrt{Y/\rho}$

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- Newton used isothermal ($\gamma = 1$): $v \approx 280$ m/s — **wrong!** Laplace corrected with adiabatic ($\gamma = 1.4$)
- $v \propto \sqrt{T}$ (Kelvin), not $\propto T$
- Speed in solids > liquids > gases (not always intuitive)

3. SUPERPOSITION & INTERFERENCE

Superposition: $y = y_1 + y_2$ (algebraic sum)
Two waves: $y_1 = A_1\sin(\omega t)$, $y_2 = A_2\sin(\omega t + \delta)$

$$A_{\text{res}} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta}$$

Constructive ($\delta = 2n\pi$): $A_{\max} = A_1 + A_2$
Destructive ($\delta = (2n+1)\pi$): $A_{\min} = |A_1 - A_2|$

$I \propto A^2$; $\dfrac{I_{\max}}{I_{\min}} = \left(\dfrac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$

Path ↔ Phase:

$$\delta = \frac{2\pi}{\lambda}\Delta x$$

Constructive: $\Delta x = n\lambda$ Destructive: $\Delta x = (n + \frac{1}{2})\lambda$

Phase change on reflection:

- Denser medium (fixed end / compression): π phase change
- Rarer medium (free end): no phase change

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- Phase change of π = path difference of $\lambda/2$ — easy to forget when combining reflected waves
- $I_{\max} \neq I_1 + I_2$ in general; energy is redistributed, not created
- Equal amplitudes ($A_1 = A_2$): $I_{\max} = 4I_0$, $I_{\min} = 0$

4. STANDING WAVES ON STRINGS

Formation: Two identical waves in opposite directions
$y_1 = A\sin(\omega t - kx)$, $y_2 = A\sin(\omega t + kx)$

$$y = 2A\cos(kx)\sin(\omega t)$$

Nodes ($A_{\text{res}} = 0$): $kx = \left(n + \frac{1}{2}\right)\pi$, separation = $\lambda/2$
Antinodes ($A_{\text{res}} = 2A$): $kx = n\pi$, separation = $\lambda/2$
Node-to-antinode distance = $\lambda/4$

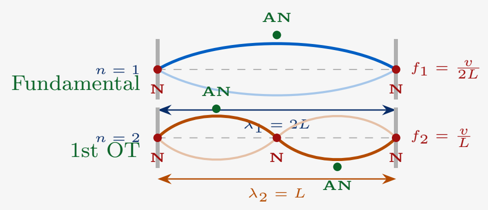
String fixed both ends:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}, \quad n = 1, 2, 3, \ldots$$

All harmonics present; f_1 = fundamental, $f_n = nf_1$

String fixed one end (free other):

$$f_n = \frac{nv}{4L}, \quad n = 1, 3, 5, \ldots \text{ (odd only)}$$



N=Node **AN**=Antinode

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- No energy transport in standing waves — energy oscillates in place
- **Displacement node = pressure antinode** (and vice versa) for sound
- n loops ⇒ n antinodes, $(n+1)$ nodes (for fixed–fixed string)

5. ORGAN PIPES

Open Pipe (antinodes at both ends):

$$f_n = \frac{nv}{2L}, \quad n = 1, 2, 3, \ldots \quad \textbf{(all harmonics)}$$

Closed Pipe (node at closed, antinode at open):

$$f_n = \frac{nv}{4L}, \quad n = 1, 3, 5, \ldots \quad \textbf{(odd only)}$$

End correction ($e \approx 0.6r$, r=pipe radius):
Open: $L_{\text{eff}} = L + 2e$ Closed: $L_{\text{eff}} = L + e$
$f_1^{\text{open}} = 2f_1^{\text{closed}}$ (same length L)

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- Closed pipe: open end = displacement **antinode** = pressure **node**; closed end = displacement **node** = pressure **antinode**
- Closed pipe misses even harmonics — produces hollow/reedy tone
- End correction cancels in resonance column experiment: $v = 2f(L_2 - L_1)$

6. BEATS

Resultant of two close-frequency waves:

$$y = 2A\cos\big(\pi(f_1 - f_2)t\big)\sin\big(\pi(f_1 + f_2)t\big)$$

$$f_{\text{beat}} = |f_1 - f_2| \qquad T_{\text{beat}} = \frac{1}{|f_1 - f_2|}$$

Carrier frequency $= (f_1 + f_2)/2$
Human ear detects beats: $f_{\text{beat}} \lesssim 10$ Hz
Tuning fork loaded (wax added $\Rightarrow f\downarrow$):

- Beats decrease $\Rightarrow$ original $f_{\text{fork}} > f_{\text{reference}}$
- Beats increase $\Rightarrow$ original $f_{\text{fork}} < f_{\text{reference}}$

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- Beat frequency is always $|f_1 - f_2|$, never $f_1 + f_2$
- Two beats per period of envelope (amplitude maxima = 2 per beat cycle)
- Loading a fork always *lowers* its frequency — use this to deduce which fork was higher

7. SOUND WAVES & DOPPLER EFFECT

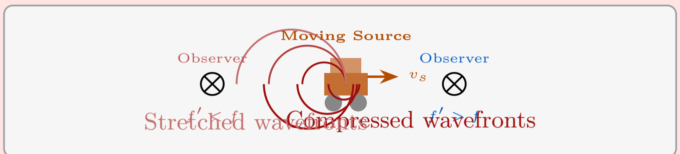
Intensity: $I = \frac{P}{4\pi r^2} \propto A^2$; $\beta = 10\log_{10}\frac{I}{I_0}$ dB, $I_0 = 10^{-12}\ \text{W/m}^2$

Doppler Formula:

$$f' = f\,\frac{v + v_o}{v - v_s}$$

- v = speed of sound in medium
- v_o = speed of observer (+ towards source)
- v_s = speed of source (+ towards observer)

Mnemonic — sign rule:
Numerator: + if observer moves *towards* source
Denominator: − if source moves *towards* observer
$\Rightarrow$ Approaching $\Rightarrow f' > f$; Receding $\Rightarrow f' < f$



Special cases:

- Both at rest: $f' = f$
- Source & observer same velocity: $f' = f$
- Source at $v_s = v$ (sonic): infinite f' ahead (shock wave)

⋆ **Student Confusion**

- **Sign convention is the most-tested trap.** Always write formula first, then assign signs — don't guess
- Doppler effect depends on *relative motion w.r.t. medium*, not just relative motion between source and observer
- Doppler effect changes *frequency*, not speed of sound
- When source moves away: $f' = f\,\frac{v}{v + v_s}$ (denominator +)

8. QUICK REVISION TABLE

Formula	Use
$v = f\lambda = \omega/k$	All waves
$v = \sqrt{T/\mu}$	String
$v = \sqrt{\gamma RT/M}$	Sound in gas
$f_n = nv/2L$	Open pipe / string
$f_n = nv/4L$ (n odd)	Closed pipe
$f' = f(v \pm v_o)/(v \mp v_s)$	Doppler
$f_b = \lvert f_1 - f_2\rvert$	Beats
$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$	Interference
$\beta = 10\log(I/I_0)$	dB level
$v_T \approx 332 + 0.61\,t$	Sound vs. temp

9. MASTER: COMMON MISTAKES SUMMARY

1. **Phase change on reflection:** π change at denser/rigid end only; no change at free/rarer end
2. **Pressure vs. displacement nodes:** In sound, displacement node = pressure antinode (they are 90 out of phase)
3. **Doppler sign convention:** v_o positive towards source, v_s positive towards observer — always
4. **Beats:** $f_{\text{beat}} = |f_1 - f_2|$, never $f_1 + f_2$
5. **Standing waves:** No net energy transfer; energy stays localised
6. **Newton vs. Laplace:** Laplace corrected by $\sqrt{\gamma}$; Newton's formula underestimates v
7. $v \propto \sqrt{T}$ **(Kelvin):** not proportional to T or $\sqrt{t}$ (Celsius)
8. **Closed pipe:** Only odd harmonics; $f_1 = v/4L$ (not $v/2L$)
9. **Intensity** $\propto 1/r^2$**:** Only for point sources; line sources $\propto 1/r$
10. **Tuning fork loading:** Adding wax always *lowers* frequency